Quantum jumps in backaction evasion measurements of a light field component

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Backaction evasion measurements of a quadrature component of the light field vacuum necessarily induce quantum jumps in the photon number. The correlation between measurement results and quantum jump events reveals fundamental nonclassical aspects of quantization.
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In optical backaction evasion quantum nondemolition measurements, the nonlinear interaction between a meter light field and the signal field is applied to obtain information about the physical properties of the signal without absorbing it[1,2]. Nevertheless the uncertainty principle requires a nonvanishing backaction on those light field variables which do not commute with the observed signal property. By studying this unavoidable backaction, fundamental insights can be gained on the quantum mechanical relationship between non-commuting properties. In particular, the field nature of photon number quantization may be investigated by correlating phase dependent field properties with the photon number of a light field state[3,4].

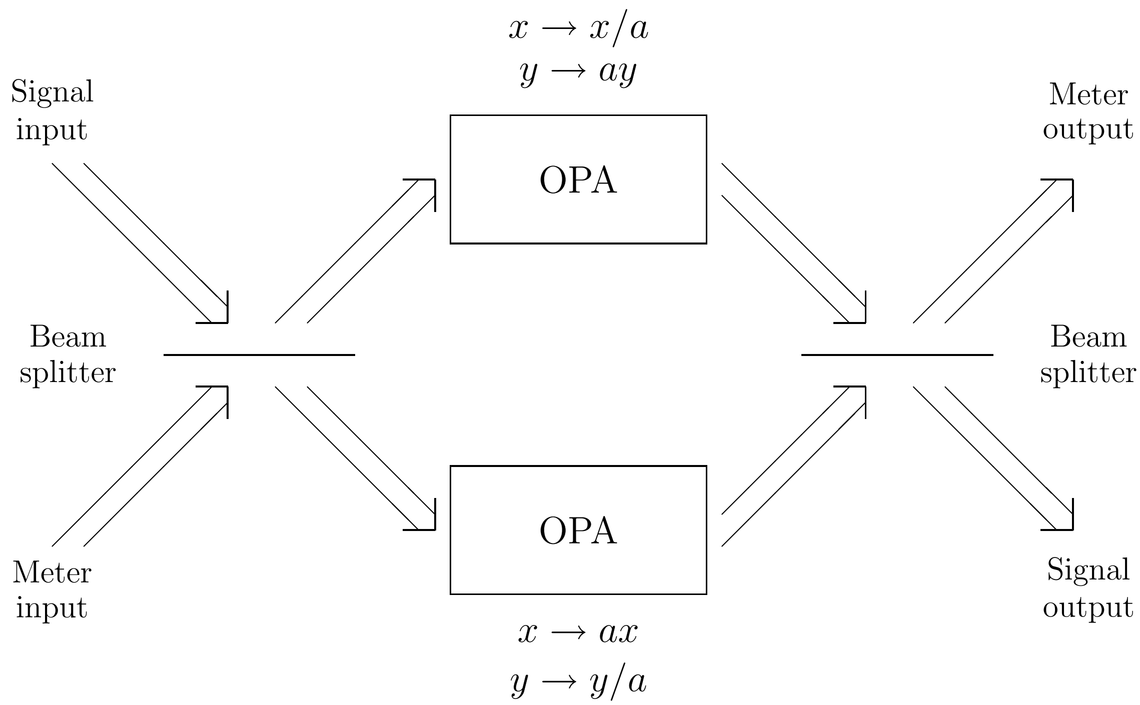

Fig. 1. Schematic setup for the quantum nondemolition measurement of the quadrature component $\hat{x}_M$ of the signal field. Note that the beam splitter reflectivity R must be adjusted to correspond to the amplification factor a of the OPAs. Its value should be equal to $R = a^2/(a^2 + 1)$.

In this presentation, we investigate the creation of photons in the vacuum by the backaction of a backaction

evasion measurement of a quadrature component of the light field. An experimental setup for such a phase sensitive backaction evasion measurement was originally proposed by Yurke[5]. A schematic representation is shown in figure 1. This measurement setup allows a measurement of the quadrature component $\hat{x}$ of the signal input. The precision of the measurement is limited by the vacuum fluctuations of the input meter field and the signal-meter coupling in the OPAs. For an amplification factor of a, the measurement resolution is given by $\delta x = a/2(a^2-1)$. The measurement statistics and the physical properties of the signal output in this setup can be described by a generalized measurement operator,

$$\hat{P}_{\delta x}(x_m) = \left(2\pi\delta x^2\right)^{-1/4} \exp\left(-\frac{(x_m-\hat{x})^2}{4\delta x^2}\right). \tag{1}$$

Using this operator, the probability distribution $P(x_m)$ of measurement results x_m and the quantum state $\mid \psi_{\mathrm{out}}(x_m)\rangle$ of the signal output for an arbitrary signal input state $\mid \psi_{\mathrm{in}}\rangle$ can be determined by

$$\begin{aligned} P(x_m) &= \langle \psi_{\mathrm{in}} \mid \hat{P}^2_{\delta x}(x_m) \mid \psi_{\mathrm{in}}\rangle \\ \mid \psi_{\mathrm{out}}(x_m)\rangle &= \frac{1}{\sqrt{P(x_m)}} \hat{P}_{\delta x}(x_m) \mid \psi_{\mathrm{in}}\rangle. \end{aligned} \tag{2}$$

The conditional final state $\mid \psi_{\mathrm{out}}(x_m)\rangle$ defines the probabilities for any further measurements performed on the signal field output. It is thus possible to determine joint probabilities and correlations between the measurement result x_m and other physical properties of the signal field.

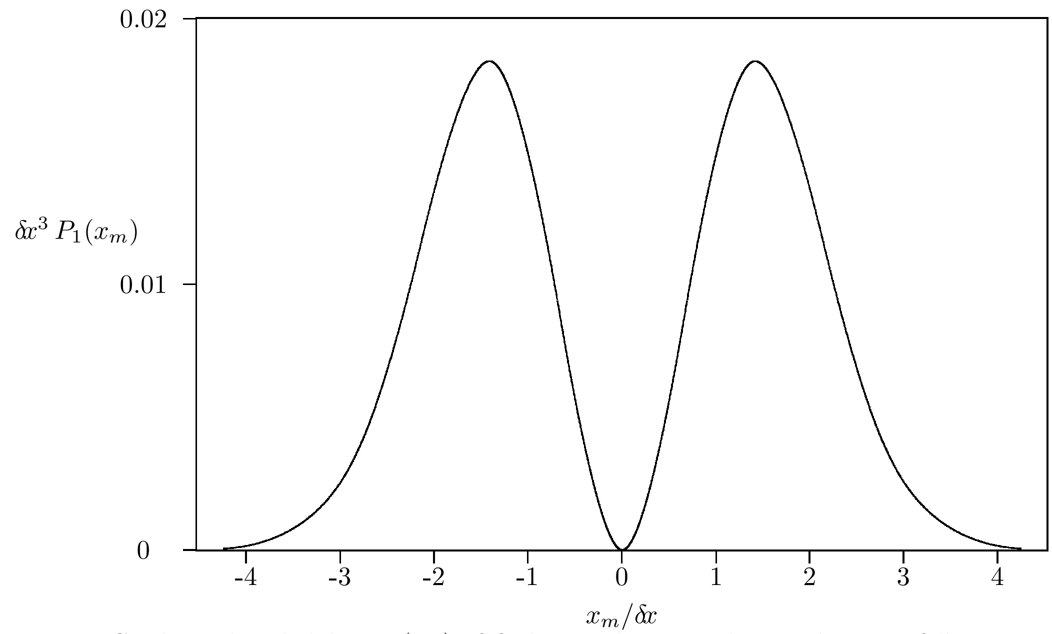

Fig. 2. Conditional probability $P_1(x_m)$ of finding a photon in the signal output following a measurement of x_m for $\delta x \gg 1$. The axes scale with δx.

In the case of a photon vacuum input state $\mid 0\rangle$, the initial photon number is zero, but even low resolution ($\delta x > 1$) measurements induce some quantum jumps to higher photon numbers. The discreteness of the quantum jump events make it difficult to understand the physical process by which the continuous field noise creates the photon. It is therefore interesting to examine the correlation between the continuous field measurement results x_m and the quantum jump events. For $\delta x > 1$, only zero and one photon contributions are relevant, so we can focus on the joint probability of finding the one photon state $\mid 1\rangle$ in the output signal after a measurement result of x_m in the meter. In the limit of high δx, this probability is given by

$$P_1(x_m) = |\langle 1 \mid \hat{P}_{\delta x}(x_m) \mid 0\rangle|^2$$
$$\approx \frac{1}{\sqrt{2\pi\delta x^2}} \frac{x_m^2}{(4\delta x^2)^2} \exp\left(-\frac{x_m^2}{2\delta x^2}\right). \tag{3}$$

As shown in figure 2, this is a double peaked probability distribution with peaks around $x_m = \pm\sqrt{2}\delta x$. The total area is equal to the quantum jump probability of $1/(16\delta x^2)$. As δx increases, the quantum jump probability decreases and the quantum jumps are associated with higher and higher field measurement results x_m. This behavior is characterized by a constant correlation between quantum jump events and the squared measurement result x_m given by

$$C(x_m^2; n) = \int dx_m P_1(x_m)\left(x_m^2 - \delta x^2\right) = \frac{1}{8}. \tag{4}$$

This constant correlation does not depend on the measurement resolution and can therefore be interpreted as a fundamental physical property of the vacuum state. In particular, an analysis of the measurement operator $\hat{P}_{\delta x}(x_m)$ reveals that this correlation originates directly from the operator ordering dependence of products of quantum variables. In terms of operator expectation values of the initial state, the quantum correlation observed in the measurement described above reads

$$C(\hat{x}^2, \hat{n}) = \frac{1}{4}\langle \hat{x}^2\hat{n} + 2\hat{x}\hat{n}\hat{x} + \hat{n}\hat{x}^2\rangle - \langle \hat{x}^2\rangle\langle \hat{n}\rangle. \tag{5}$$

Even though the vacuum state is an eigenstate of $\hat{n}$ with an eigenvalue of zero, the operator correlation $C(\hat{x}^2, \hat{n})$ is nonzero because in the term $\langle \hat{x}\hat{n}\hat{x}\rangle$ the photon number operator is sandwiched between the quadrature component operators. The (mathematical) action of $\hat{x}$ on $\mid 0\rangle$ creates a one photon state $0.5 \mid 1\rangle$. This creation of a one photon contribution is responsible for an operator correlation of $C(\hat{x}^2, \hat{n}) = 1/8$ in the vacuum state.

The presence of a correlation between quadrature components and photon number in an eigenstate of photon number raises fundamental questions about the physical meaning of eigenvalues. Obviously, an eigenvalue is not just an "element of reality" as suggested in many interpretations of quantum mechanics. The experimental observation of correlations between the quantum jumps and the quadrature component measurement results could thus provide a new perspective on the nature of quantum mechanical reality.

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